PriMo: Coupled Prisms for Intuitive Surface Modeling

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Surface Deformation

• Requirements
  – Easy and intuitive user interaction
  – Large-scale deformations
  – Robustness
  – Efficiency
Surface Deformation

- Recent methods focus more on efficiency
  - Real-time deformations of large models

- Requires linearization
  - Problems with large deformations

- Split large deformations
  - Specify more constraints
  - More user guidance required
Non-Linear Surface Deformation

• Use a non-linear deformation model
  – Too slow, complicated, instable?

• Physically *plausible* vs. physically *correct*

• Trade physical correctness for
  – Computational efficiency
  – Numerical robustness
Comparison

Original

VarMin

Grad

PriMo
Outline

• Motivation
• Prism Representation
• Geometric Optimization
• Results
Elastically Connected Rigid Prisms

- Qualitatively emulate thin-shell behavior
- Thin volumetric layer around center surface
- Extrude polygonal prism $P_i$ per mesh face $F_i$
• How to deform prisms?
  – FEM has problems if elements degenerate...

• Prevent prisms from degenerating
  ➡ Keep them \textit{rigid}
Elastically Connected Rigid Prisms

- Connect prisms along their faces
  - Non-linear elastic energy
  - Measures bending, stretching, twisting, ...
Elastically Connected Rigid Prisms

- Pairwise prism energy

\[ E_{ij} = \int_{[0,1]^2} \| f^{i\rightarrow j}(u) - f^{j\rightarrow i}(u) \|^2 du \]
Physical Interpretation

\[ E_{ij} = \int_{[0,1]^2} \left\| f^{i\rightarrow j}(u) - f^{j\rightarrow i}(u) \right\|^2 du \]

Integral over infinitesimal spring fibres

Sum of spring energies

\[ E_{ij} \approx \sum_k \left\| f^{i\rightarrow j}_k - f^{j\rightarrow i}_k \right\|^2 \]
Elastically Connected Rigid Prisms

- **Pairwise prism energy**

\[
E_{ij} = \int_{[0,1]^2} \| f^{i\to j}(u) - f^{j\to i}(u) \|^2 \, du
\]

- **Global energy**

\[
E = \sum_{\{i,j\}} w_{ij} \cdot E_{ij}, \quad w_{ij} = \frac{\| e_{ij} \|^2}{|F_i| + |F_j|}
\]
Prism-Based Surface Deformation

1. Prescribes position/orientation for prisms
2. Find optimal rigid motions per prism
3. Update vertices by average prism transformations
Outline

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Non-Linear Minimization

- Find rigid motion \((R_i, t_i)\) per prism \(P_i\)

\[
\min_{\{R_i, t_i\}} \sum_{\{i,j\}} w_{ij} \int_{[0,1]^2} \| R_i f^{i \rightarrow j}(u) + t_i - R_j f^{j \rightarrow i}(u) - t_j \|^2 du
\]
Continuous Shape Matching

$$\min \left\{ R_i, t_i \right\} \sum_{i,j} w_{ij} \int_{[0,1]^2} \left\| R_i f^{i\rightarrow j}(u) + t_i - R_j f^{j\rightarrow i}(u) - t_j \right\|^2 du$$

$\min \{ R_i, t_i \} \sum_{i,j} w_{ij} \sum_k \left\| R_i f^{i\rightarrow j}_k + t_i - R_j f^{j\rightarrow i}_k - t_j \right\|^2 du$
Non-Linear Minimization

• Find rigid motion \((R_i, t_i)\) per prism \(P_i\)

\[
\min_{\{R_i, t_i\}} \sum_{\{i,j\}} w_{ij} \int_{[0,1]^2} \| R_i f^{i\rightarrow j}(u) + t_i - R_j f^{j\rightarrow i}(u) - t_j \|^2 \, du
\]

• Generalized shape matching problem
  – Discrete point correspondences vs. continuous face correspondences

⇒ Adapt techniques for point-set registration
Iterated Local Shape Matching

• Iterate this:
  – Randomly pick one prism
  – Optimize its position/orientation [Horn87]

\[
\min_{R_i, t_i} \sum_{j \in N_i} w_{i,j} \int_{[0,1]^2} \| R_i f^{i \rightarrow j}(u) + t_i - f^{j \rightarrow i}(u) \|^2 \, du
\]

• Corresponds to error diffusion
  – Rapidly removes high error frequencies
  – Impractically slow convergence
Global Shape Matching  [Pottmann 04]

- First order approx. of rigid motions
  \[ \mathbf{R}_i (\cdot) + \mathbf{t}_i \approx (\cdot) + \omega_i \times (\cdot) + \mathbf{v}_i =: A_i (\cdot) \]

- Quadratic minimization wrt. velocities
  \[ \min_{\{\mathbf{v}_i, \omega_i\}} \sum_{\{i,j\}} w_{ij} \int_{[0,1]^2} \| A_i (f^{i\rightarrow j} (u)) - A_j (f^{j\rightarrow i} (u)) \|^2 du \]

- Yields affine motion \( A_i \) per prism
  - Project to manifold of rigid motions
Global Shape Matching

• Find “closest” rigid motion
  – Measure distance of transformations’ images
  – Another local shape matching

• Larger steps, fewer iterations
  – Factor 50 faster than [Pottmann02]
Global Shape Matching

while not converged
{
    find optimal velocities \([v_i, w_i]\)
    \(\forall i: (R_i, t_i) = \text{project}(v_i, w_i)\)
    \(\forall i: P_i = R_i P_i + t_i\)
}

Performance: \(~7k\) prism updates per second
Hierarchical Shape Matching

- Local and global matching alone don’t work
  - Slow convergence of local matching
  - High complexity of global matching

- Hierarchical multi-grid matching
  - Solve global matching on coarse level
  - Apply local matching on finer levels
Robustness
Robustness
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Dragon Deformation

VarMin  RBF  Grad  RotInv  PriMo
Prism Parameters

Original  Height  Width  Angle--  Angle++
Stiffness Control

- Height
- Width
- Angle--
- Angle++
Control Surface Area

- Height
- Width
- Angle--
- Angle++
Non-Shrinking Smoothing

Height

Width

Angle--

Angle++
Detail Enhancement

- Height
- Width
- Angle--
- Angle++
Force-Based Deformation

- Separately prescribe
  - positions and/or
  - orientations

- Forces can be more intuitive
  - Physically intuitive
  - Constraints = high forces

- Incorporate forces
  - Just another spring energy
Force-Based Deformation
Goblin Posing
Goblin Posing

- Decreased stiffness at joints
- Forces & hard constraints
- 180k triangles at about 1 fps
- Whole session < 5 min
Conclusion

- Non-linear surface deformation model
  - Physically plausible
  - Intuitive parameters for surface behavior
  - Constraint-based and force-based

- Hierarchical shape matching
  - Extremely robust
  - Reasonably efficient
  - Easy implementation